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Distribution System Analysis

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Distribution System Analysis

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1. Introduction

- The objective of distribution system analysis is to determine the state of the system including voltages, real and reactive power flow on lines, and losses in the system.

- This requires modeling all the components in the system such as lines and transformers and their interconnections based on the topology. In addition, models for loads and sources connected to the system are needed.

- Since distribution system is connected to transmission systems, which are connected to large generators, we model the whole upper level system at the point of coupling as an equivalent source.

- Distributed energy resources (DERs) directly connected to the distribution system are modeled based on their characteristics.

2. Modeling of Source Impedance

- A method that is valid for both radial and loop systems is to derive the sequence impedance values from the results of a fault study.
- The procedure involves considering different types of faults at the bus of interest and using the equivalent circuit for each fault to determine the source impedance for positive, negative, and zero sequences, as given below for bus *m.*

$$Z_1^{sm} = \frac{V}{I_\beta^m} - Z_f$$

$$Z_2^{sm} = \frac{j\sqrt{3}V}{I_{fLL}^m} - Z_{s1}^m - Z_f$$

$$Z_0^{sm} = \frac{3V}{I_{fLG}^m} - Z_{s1}^m - Z_{s2}^m - 3Z_f$$

Subscripts 1,2,and 0 represents positive, negative and zero sequence quantities

2. Modeling of Source Impedance

Where

$\boldsymbol{V}$, system nominal voltage (line neutral);

$\boldsymbol{I_{\beta\mathcal{M}}^{m}}$, three-phase fault current at bus m;

$\boldsymbol{I_{fLL}^{m}}$, line-to-line fault current at bus m;

$\boldsymbol{I_{fLG}^{m}}$, SLG fault current at bus m; $\boldsymbol{Z_{f}}$, fault impedance

- This method requires the presence of all three phases, as well as line-to-line and single-line-to-ground faults, so it can only be used on buses with all three phases.

2. Modeling of Source Impedance

- Knowing the sequence values, the corresponding values in the phase domain can be obtained by the proper symmetrical component similarity transformation.
- However, the obtained result will be an approximation because all the lines must be fully transposed to balance the three phases for decoupling of the sequence impedances.
- This assumption works well in transmission systems but not for distribution systems. So, if we have to represent the entire transmission system at the substation, this approach will work well.

Approximate Line Segment Model

- The approximate line model can be developed by applying the "reverse impedance transformation" from symmetrical component theory.
- Using the known positive and zero sequence impedances, the "sequence impedance matrix" is given by (See Chapter 4, Kersting)

$$[Z_{seq}] = \begin{bmatrix} Z_0 & 0 & 0 \\ 0 & Z_+ & 0 \\ 0 & 0 & Z_+ \end{bmatrix} \qquad (47)$$

- Note that this assumption means lines are assumed to be transposed, that is why this model is called "approximate" model.
- The "reverse impedance transformation" results in the following approximate phase impedance matrix:

$$[Z_{approx}] = [A_s] \cdot [Z_{seq}] \cdot [A_s]^{-1} \qquad (48)$$

$$[Z_{approx}] = \frac{1}{3} \cdot \begin{bmatrix} (2Z_+ + Z_0) & (Z_0 - Z_+) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (2Z_+ + Z_0) & (Z_0 - Z_+) \\ (Z_0 - Z_+) & (Z_0 - Z_+) & (2Z_+ + Z_0) \end{bmatrix} \qquad (49)$$

2. Modeling of Source Impedance

- For loop systems, if $[Z_{BUS}]$ matrix of the entire system in phase domain is known, the diagonal submatrix $\left[Z_{\text{abc}}^{m}\right]$ corresponding to bus m is the source impedance (3×3) matrix. This is also called the driving point impedance matrix.

$$[Z_{abc}^{m}] = \begin{bmatrix} Z_{aa}^{sm} & Z_{ab}^{sm} & Z_{ac}^{sm} \\ Z_{ab}^{sm} & Z_{bb}^{m} & Z_{bc}^{sm} \\ Z_{ac}^{sm} & Z_{bc}^{s} & Z_{cc}^{sm} \end{bmatrix}$$

Note: the driving point impedance obtained based on the positive-sequence network topology of the system should not be used for fault calculations because it does not account for transformer connections that are important for the zero-sequence network.

3. Load Models

- Under steady state, complex power **S** at any location in a distribution system varies with voltage and can be described as a function of voltage V at that point, that is

$$\boldsymbol{S} = \boldsymbol{V}\mathrm{I}^* = P + jQ = f(\boldsymbol{V})$$

- Different modeling approaches are used to describe the relationship of the above equation.

3. Load Models

3.1 Load Model I

- It is usual to represent both P and Q as a general polynomial function of V.

$$P = a_0 + a_1 V + a_2 V^2 + a_{-1} V^{-1} + a_{-2} V^{-2} + \cdots$$
$$Q = b_0 + b_1 V + b_2 V^2 + b_{-1} V^{-1} + b_{-2} V^{-2} + \cdots$$

- Such a representation is valid for individual type of loads or aggregate (composite) type of loads.
- Different conditions for the coefficients give different models. For example:
  i. If only a_0 and b_0 are nonzero, and all the coefficients are zero, then,

$P = a_0$ and $Q = b_0$ **(Constant Power Load)**

3. Load Models

ii. If only a_1 and b_1 are nonzero, and all the coefficients are zero, then,

$$P = a_1 V \text{ and } Q = b_1 V$$ **(Constant Current Load)**

iii. If only a_2 and b_2 are nonzero, and all the coefficients are zero, then,

$$P = a_2 V^2 \text{ and } Q = b_2 V^2$$ **(Constant Impedance Load)**

3. Load Models

- If a load is a combination of the three above mentioned types, we can combine them to find a composite expression. For examples, tests and subsequent regression analysis on the data show the following models for common load types:

(a) Air-conditioning load demand (per-unit values):

$$P = 2.97 - 4.00V + 2.02V^2$$

$$Q = 12.90 - 26.8V + 14.90V^2$$

(b) Fluorescent lighting:

$$P = 2.18 + 0.286V - 1.45V^{-1}$$

$$Q = 6.31 - 15.60V + 10.30V^2$$

(c) Induction motor:

$$P = 0.720 + 0.109V + 0.172V^{-1}$$

$$Q = 2.80 + 1.63V - 7.60V^2 + 4.89V^3$$

3. Load Models

3.2 Load Model II

- Composite loads, which are assumed to be mixtures of the types as discussed, can be represented as:

$$P = P_n \left(\frac{V}{V_n}\right)^k$$

$$Q = Q_n \left(\frac{V}{V_n}\right)^l$$

where both k and l vary between 0 and 3, V_n is the initial or base value of voltage, P_n is the initial or base value of real power, and Q_n is the initial or base value of reactive power. When using these models, we should be aware of the range of V for the models to be valid.

3. Load Models

3.2 Load Model II

Some examples of using Load Model II:

- (a) If k = 1 and l = 0, it implies that the load is a constant current type with unity power factor.
- (b) If k = l = 2, the load is a constant impedance type.
- (c) If k = 2.5 and l = 2.7, it represents an aluminum reduction plant. This is a simple model and can be determined empirically from measurements.

3. Load Models

3.3 Load Model III

- If loads (or demands) are sensitive to frequency, the frequency effects should be included. In that case, voltage and frequency dependence are described by the following relationships:

$$P(f,V) = P_n \left(\frac{\omega}{\omega_n}\right)^{\alpha} \left(\frac{V}{V_n}\right)^{k}$$

$$Q(f,V) = Q_n \left(\frac{\omega}{\omega_n}\right)^{\beta} \left(\frac{V}{V_n}\right)^{l}$$

where $\omega = 2\pi f$, $\omega_n = 2\pi f_n$, f_n is the base frequency, and α and β are constant exponents.

3. Load Models

3.3 Load Model III

- Instead of determining the exponent for this model, it is often a practice to determine the four sensitivity coefficients. Knowing these coefficients, the new values for ΔP and ΔQ can be determined from Δf and ΔV. All the changes are assumed to be small, thus:

$$\begin{bmatrix} \Delta P \\ \Delta Q \end{bmatrix} = \begin{bmatrix} \frac{\partial P}{\partial f} & \frac{\partial P}{\partial V} \\ \frac{\partial Q}{\partial f} & \frac{\partial Q}{\partial V} \end{bmatrix} \begin{bmatrix} \Delta f \\ \Delta V \end{bmatrix}$$

3. Load Models

3.3 Load Model III

- The table shows the sensitivities coefficients based on the results of a data survey conducted by Electric Power Research Institute (EPRI). The values have been normalized to apparent power, S.

- Then, $P = Pn + \Delta P$ and $Q = Qn + \Delta Q$ for $f = fn + \Delta f$ and $V = Vn + \Delta V$

Table: Suggested values of sensitivities of real power and reactive power to voltage and frequency changes based on load survey results.

Load type	$\frac{\Delta P/S}{\Delta V}$	$\frac{\Delta Q/S}{\Delta V}$	$\frac{\Delta P/S}{\Delta f}$	$\frac{\Delta Q/S}{\Delta f}$
Composite load	0.7–1.2	1.0–2.0	0.6–1.5	0 to –0.6
Residential	1.0–1.5	1.0–1.4	0.5	–0.7
Commercial	1.2	1.17	–0.185	–0.488
Industrial	0.7–1.5	1.0–2.0		

3. Load Models

3.4 Load Model IV

- The model is particularly suitable for modeling uncertainties in aggregate loads at a node knowing the demand profiles for a day, a season, or a year.
- Assuming Gaussian distribution for load demand:

$$P_p = P_n + k_p\sigma \text{ , and } Q_p = Q_n + k_p\sigma$$

where, P_p is the power value at which the probability of load exceeding that value is p%, k_p is the coefficient related to p, and σ is the standard deviation of the load.

- Typically, p of 10% is used in voltage-drop calculations, and p of 50% or mean values of load are used for loss calculations. Smaller values of p are used for overload and determination of emergency conditions.

4. Distributed Energy Resources (DERs)

- Traditional distribution systems were not designed to accommodate active generation and storage. However, with decreasing cost and advances in technology, such devices are being deployed in distribution systems.
- DERs are defined as sources of electric power that are not directly connected to the bulk power system but are connected to the distribution system, limited in size to 10 MVA or less.
- DER includes generators of different types and energy storage devices with the ability to inject power into the system.

4. Distributed Energy Resources (DERs)

- Generators include rotating induction or synchronous rotating machines driven by burning diesel, natural gas, bio gas, propane, or by wind or water flow. The static types of DERs include solar photovoltaic (PV) and batteries.
- The majority of DERs are connected to the system through a power electronic interface:
  - For the resources that generate AC power, the converter changes it to DC power, and an inverter changes it back to AC power at the system frequency.
  - For the resources that produce DC power, the inverter changes it to AC power at the system frequency.
- The DERs that use a power electronics controller with embedded inverters are called inverter-based resources (IBRs).

4. Distributed Energy Resources (DERs)

Impacts of DERs on Voltage Regulation

- Addition of DERs can impact voltage in distribution systems.
- Previously, DERs were not permitted to actively regulate voltage. They operated at a fixed power factor.
- IEEE 1547 standard revised in 2018 permits DERs to regulate voltage by injecting reactive power.

4. Distributed Energy Resources (DERs)

Impacts of DERs on Voltage Regulation

- DERs are divided into Category A and Category B.
  - Category A for the systems with lower penetration of DERs.
  - Category B for the systems with higher penetration of DERs or system with frequent large variations in power output.
- The required reactive power capabilities of the DERs for Category A and Category B are shown in the Figure. These requirements apply to DERs for continuous operation when the voltage is 0.88 and 1.1 times the nominal voltage.

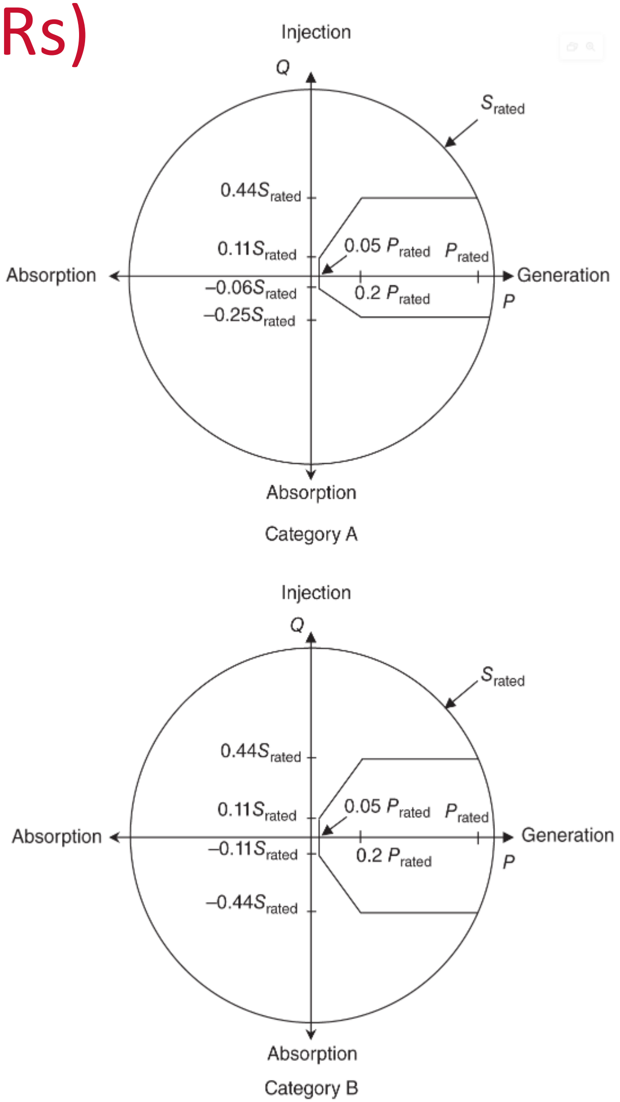


Fig: Minimum reactive power capability of Category A and B DER.

4. Distributed Energy Resources (DERs)

Control Function Requirements for DERs

- Although constant power factor mode with unity power factor setting is the default mode for DER operation unless otherwise specified by the distribution system operator, the distribution system operator has to approve active participation of DERs in voltage regulation.
- The 1547 standard requires DERs to have the ability to control voltage, reactive power, and real power within the operating region. These control function requirements are specified in the Table.

Control mode	Category A	Category B
Constant power factor	Mandatory	Mandatory
Volt-var	Mandatory	Mandatory
Watt-var	Not required	Mandatory
Constant reactive power	Mandatory	Mandatory
Volt–watt	Not required	Mandatory

Table: Voltage, reactive power, and real power control function requirements for DER

Control Function Requirements for DERs

- Control modes are illustrated in the figures as per the IEEE Standard 1547.

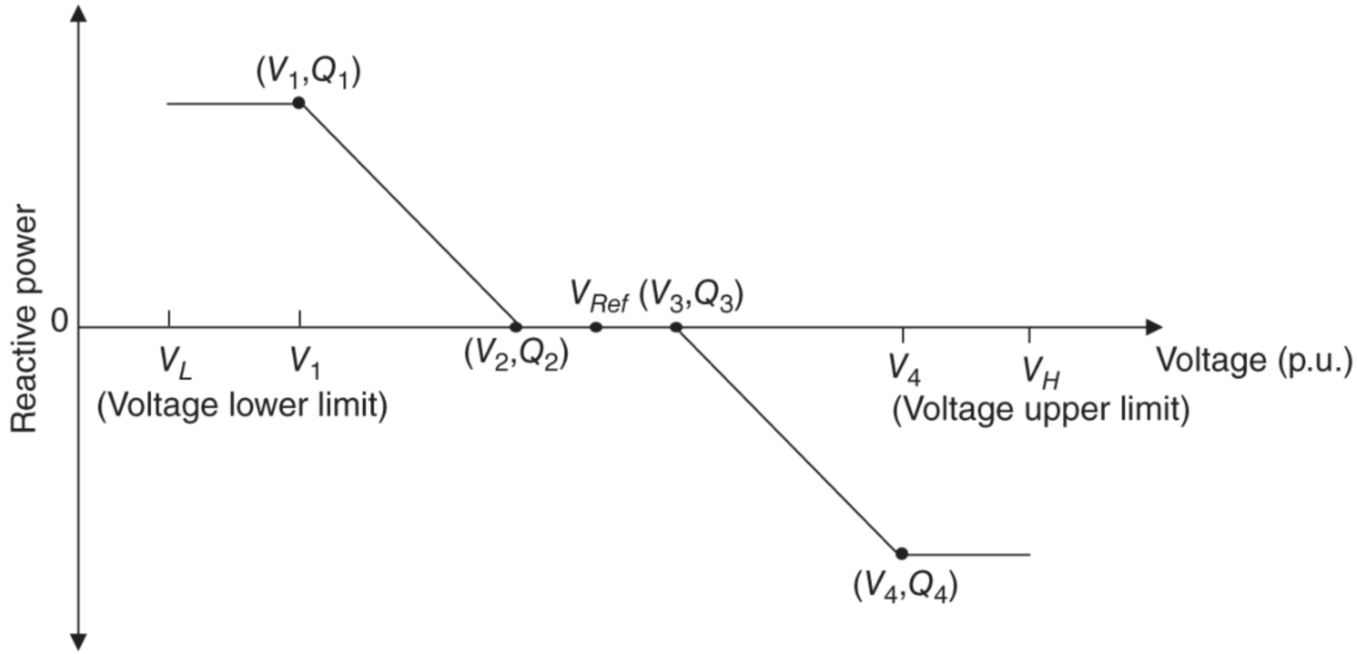


Fig: Volt-VAr characteristic for DER control

Control Function Requirements for DERs

- Control modes are illustrated in the figures as per the IEEE Standard 1547.

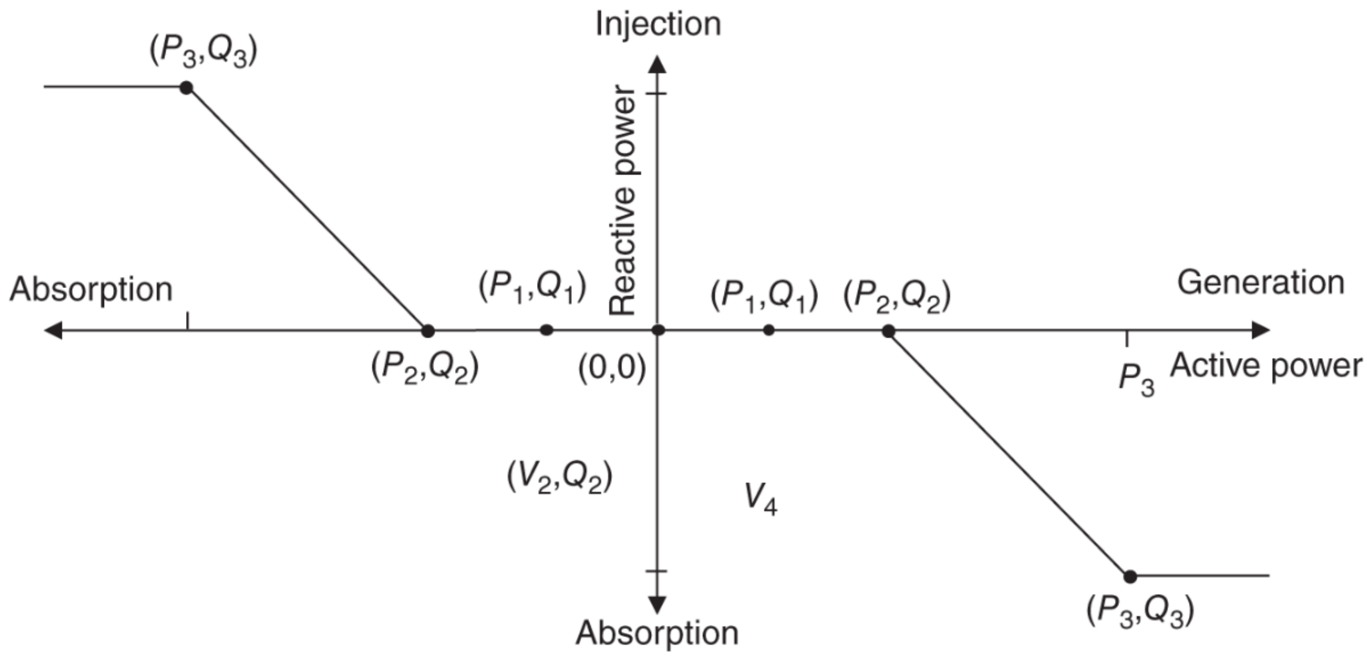


Fig: Watt-VAr characteristic for DER control

Control Function Requirements for DERs

- Control modes are illustrated in the figures as per the IEEE Standard 1547.

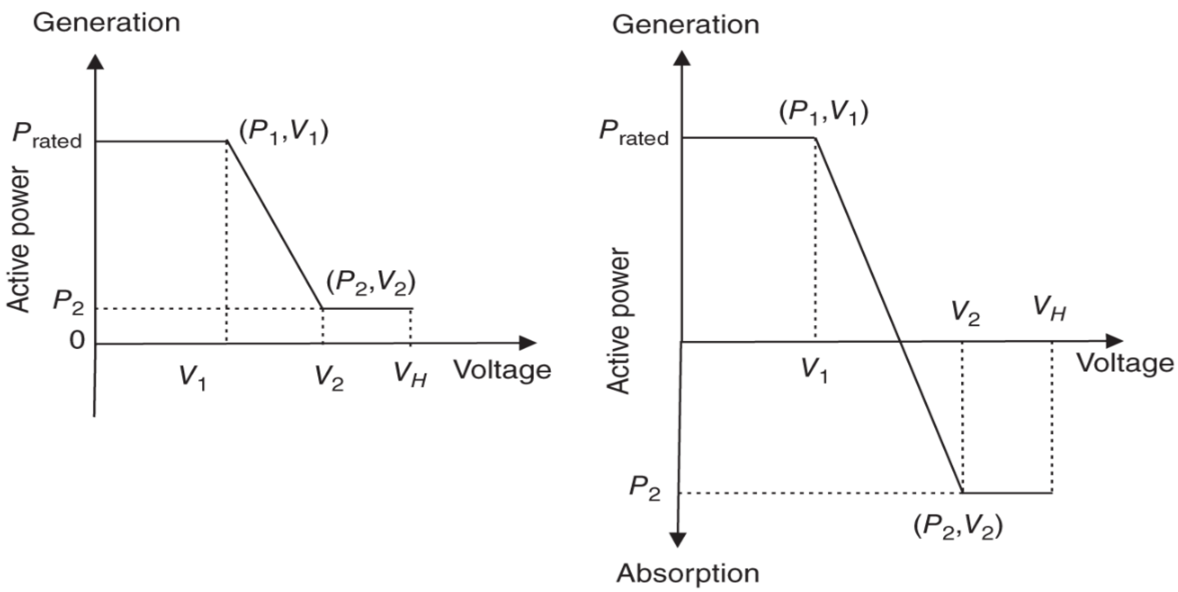


Fig: Volt–Watt characteristic for DER control.

Control Function Requirements for DERs

- Standard 1547 provides suggested values for various set points shown in these figures.
- The DERs will provide the capabilities of mutually exclusive reactive power control modes listed in this table and will be capable of initiating any of these modes one at a time.

Control Function Requirements for DERs

- DER operator is responsible for implementing changes to settings and mode selections upon request by the system operator within a specified time. Other control modes mutually agreeable to the DER operator and the system operator can also be implemented.
- Irrespective of the type of DER and the selected control mode, DERs supply real power and either supply or absorb reactive power.
- Thus, while considering DER as a load, one must consider its real power and reactive power as negative if delivering it, and positive if absorbing it.

5. Power Flow Studies

- Since a typical distribution system is unbalanced due to both design and operation, it is imperative that the system's steady-state performance evaluation or analysis be conducted in three-phase domain, or to consider each phase separately
- It is unlike transmission systems, where usually single-phase power flow studies are conducted while considering the system to be balanced.

5. Power Flow Studies

- To facilitate distribution system analysis, various models shown in the table are used for the components of the system.

Components	Mathematical Model
Substation	Infinite source as a reference bus, where voltage magnitude can be controlled using regulators and/or taps on the transformers
Feeders and Laterals	Three-phase series impedance and shunt admittance matrices for each line section. Shunt admittance matrix is included for long underground cable with significant charge currents
Load	Complex power $(P_i^{abc} + jQ_i^{abc})$ for each bus *i*
DER	Complex power $(P_i^{abc} + jQ_i^{abc})$ for each bus *i*

5. Power Flow Studies

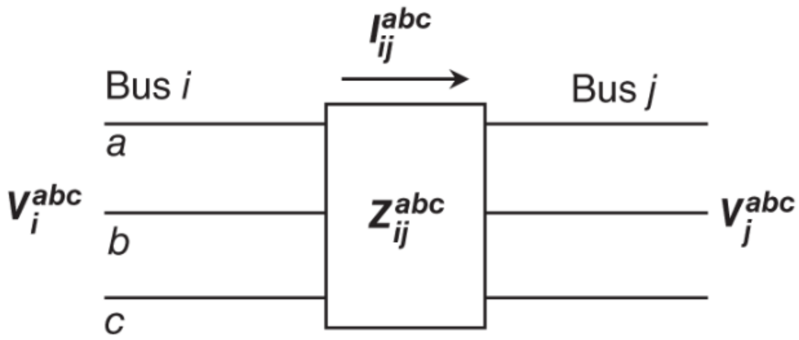


Fig: Schematic of a three-phase line connected between buses *i* and *j*

5.1 Line Model

- The above figure shows the schematic of a three- phase line connected between buses *i* and *j*.
- For simplicity, only the series impedance of the line (Z_{ij}^{abc}) is shown.
- V_i^{abc} and V_j^{abc} is the vector of phase *a*, *b*, and *c* voltages at bus *i* and bus *j* respectively,
- I_{ij}^{abc} is the vector of currents flowing in phases *a*, *b*, and *c* voltages from bus *i* to *j* of the line.

5. Power Flow Studies

5.1 Line Model

- Then, we have the expressions as,

$$V_{ij}^{abc} = Z_{ij}^{abc} \cdot I_{ij}^{abc}$$
$$V_{ij}^{abc} = V_i^{abc} - V_j^{abc}$$

- Z_{ij}^{abc} is a 3 *X 3* matrix, which is realized after reducing a fourth-order model of the line by Kron's reduction.
- For a two-phase line, Z_{ij}^{abc} matrix has nonzero entries in the 2×2 submatrix corresponding to the phases of the line.
- For a single-phase line, it has nonzero entry in the diagonal entry corresponding to the phase of the line.

5. Power Flow Studies

5.2 Load and DER Model

Y-Connected:

- For modeling loads and DER (negative load), we consider that the real and reactive power for each of the three phases are known separately.

- For a three-phase Y-connected load, the complex power drawn is shown in the Figure. For this load connected at bus *i,* as shown in the figure, the vector S_i^{abc} of complex power is given by:

$$\boldsymbol{S}_i^{abc} = \begin{bmatrix} \boldsymbol{S}_i^a \\ \boldsymbol{S}_i^b \\ \boldsymbol{S}_i^c \end{bmatrix} = \begin{bmatrix} P_i^a + jQ_i^a \\ P_i^b + jQ_i^b \\ P_i^c + jQ_i^c \end{bmatrix}$$

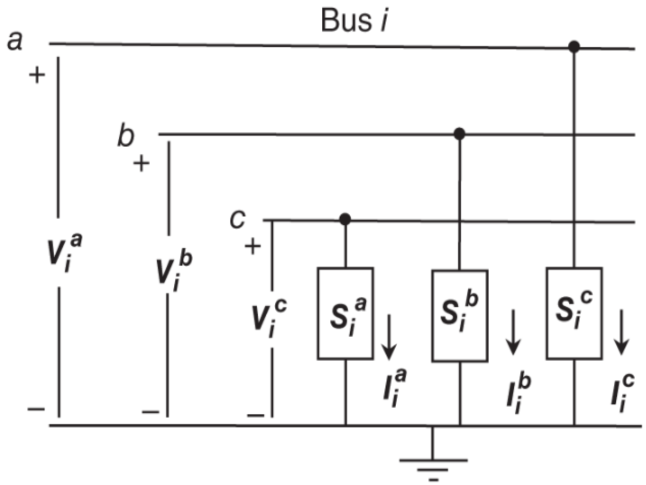


Fig: Representation of a three-phase Y-connected Load on bus *i*

5. Power Flow Studies

- The vectors of voltages and currents at bus *i* are given by,

$$V_i^{abc} = \begin{bmatrix} V_i^a \\ V_i^b \\ V_i^c \end{bmatrix}, \text{ and } I_i^{abc} = \begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}$$

- Now, consider a matrix operation UV_i^{abc}, which gives the following matrix:

$$UV_i^{abc} = \begin{bmatrix} V_i^a & 0 & 0 \\ 0 & V_i^b & 0 \\ 0 & 0 & V_i^c \end{bmatrix}$$

Hence,

$$\begin{bmatrix} S_i^a \\ S_i^b \\ S_i^c \end{bmatrix} = \begin{bmatrix} \boldsymbol{V}_i^a & 0 & 0 \\ 0 & V_i^b & 0 \\ 0 & 0 & V_i^c \end{bmatrix} \begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}^*$$

5. Power Flow Studies

And,

$$\begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}^* = \begin{bmatrix} \boldsymbol{V}_i^a & 0 & 0 \\ 0 & V_i^b & 0 \\ 0 & 0 & V_i^c \end{bmatrix}^{-1} \begin{bmatrix} S_i^a \\ S_i^b \\ S_i^c \end{bmatrix}$$

- Note that for two- or single-phase loads, S_i^{abc}, V_i^{abc} , and I_i^{abc} must be truncated by removing the entries for phases that do not exist in the load.

5. Power Flow Studies

Delta- Connected:

- Consider a Δ-connected load as shown in the figure.
- A relationship between complex powers, voltages, and currents is,

$$\begin{bmatrix} S_i^{ab} \\ S_i^{bc} \\ S_i^{ca} \end{bmatrix} = \begin{bmatrix} V_i^{ab} & 0 & 0 \\ 0 & V_i^{bc} & 0 \\ 0 & 0 & V_i^{ca} \end{bmatrix} \begin{bmatrix} I_i^{ab} \\ I_i^{bc} \\ I_i^{ca} \end{bmatrix}^*$$

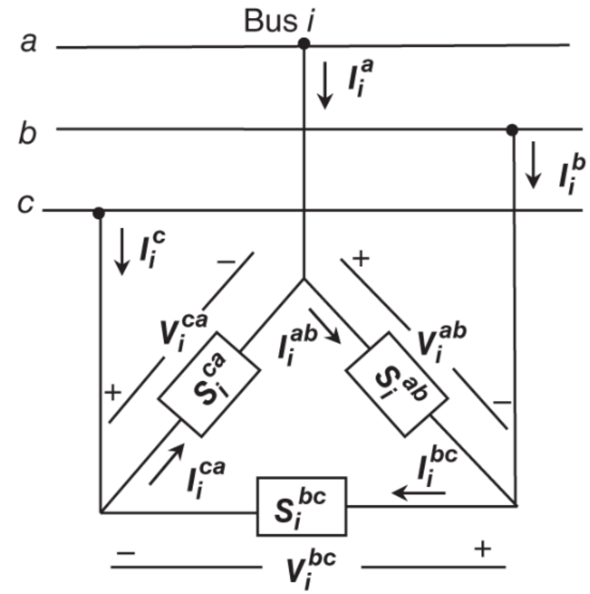


Fig: Representation of a three-phase Δ-connected load on bus i.

5. Power Flow Studies

And,

$$\begin{bmatrix} I_i^{ab} \\ I_i^{bc} \\ I_i^{ca} \end{bmatrix}^* = \begin{bmatrix} V_i^{ab} & 0 & 0 \\ 0 & V_i^{bc} & 0 \\ 0 & 0 & V_i^{ca} \end{bmatrix}^{-1} \begin{bmatrix} S_i^{ab} \\ S_i^{bc} \\ S_i^{ca} \end{bmatrix}$$

- Following that, we can compute $\boldsymbol{I}_{ij}^{abc}$ using Kirchhoff's current law (KCL),

$$I_i^a = I_i^{ab} - I_i^{ca}$$
$$I_i^b = I_i^{bc} - I_i^{ab}$$
$$I_i^c = I_i^{ca} - I_i^{bc}$$

5. Power Flow Studies

5.3 Computing Currents

- Knowing the load or demand vectors $\boldsymbol{S}^{abc}$ at each bus, the corresponding current vectors $\boldsymbol{I}^{abc}$ can be found using following equation given the voltage vectors $\boldsymbol{V}^{abc}$ at the buses.

$$\begin{bmatrix} I_i^a \\ I_i^b \\ I_i^c \end{bmatrix}^* = \begin{bmatrix} \boldsymbol{V}_i^a & 0 & 0 \\ 0 & V_i^b & 0 \\ 0 & 0 & V_i^c \end{bmatrix}^{-1} \begin{bmatrix} S_i^a \\ S_i^b \\ S_i^c \end{bmatrix}$$

5. Power Flow Studies

5.3 Computing Currents

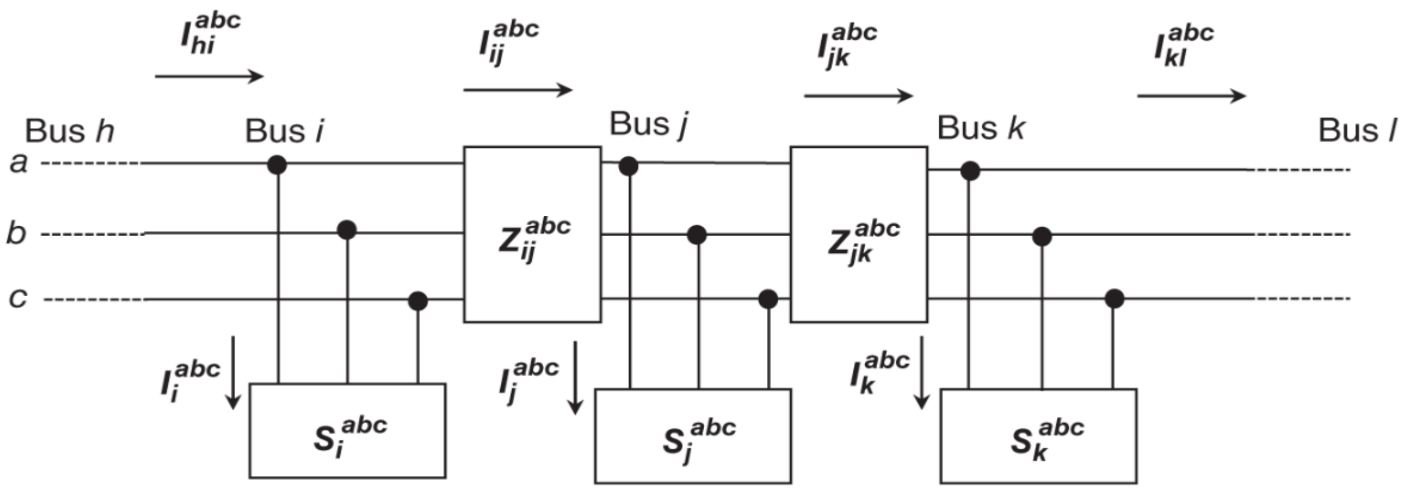
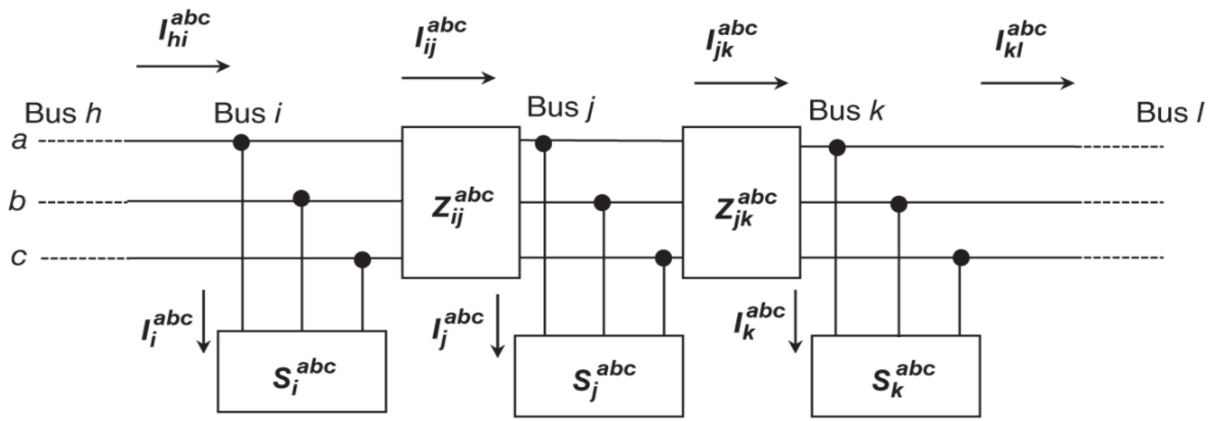


Fig: Schematic showing line sections and connected loads.

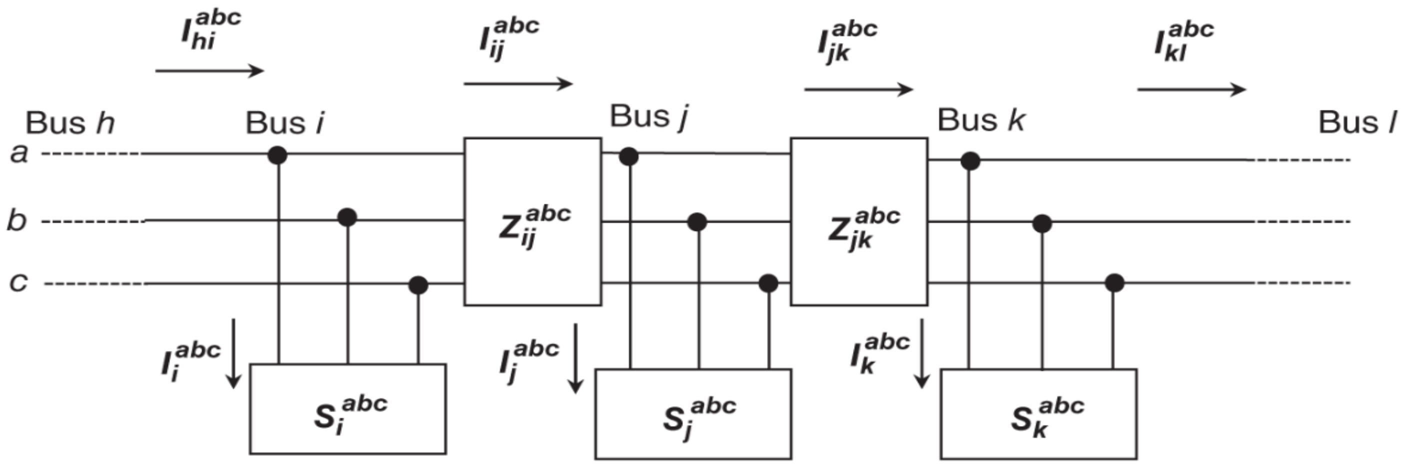
- The above figure shows two three-phase feeders connected between buses *i, j,* and *k* with respective loads at these buses.

5. Power Flow Studies



- $\boldsymbol{I_i^{abc}}$, $\boldsymbol{I_j^{abc}}$, and $\boldsymbol{I_k^{abc}}$ are load current vectors at buses *i, j,* and *k*, respectively.
- $\boldsymbol{I_{ij}^{abc}}$ and $\boldsymbol{I_{jk}^{abc}}$ are current vectors of currents flowing on lines between buses *i* and *j*, and *j* and *k*.
- $\boldsymbol{I_{hi}^{abc}}$ is vector of currents entering bus *i* from bus *h*, and $\boldsymbol{I_{kl}^{abc}}$ is vector of currents leaving bus *k* toward bus *l*.

5. Power Flow Studies



- Applying KCL at buses, *i*, *j*, and *k*, we get,

$$I_{hi}^{abc} = I_{ij}^{abc} + I_i^{abc}$$
$$I_{ij}^{abc} = I_{jk}^{abc} + I_j^{abc}$$
$$I_{jk}^{abc} = I_{kl}^{abc} + I_k^{abc}$$

5. Power Flow Studies

- Similar equations can be written for additional sections in the system. If there are additional feeders splitting off of the main feeder, the KCL can be expanded. For example, consider a case where two feeders are splitting at bus k as shown in the Figure:

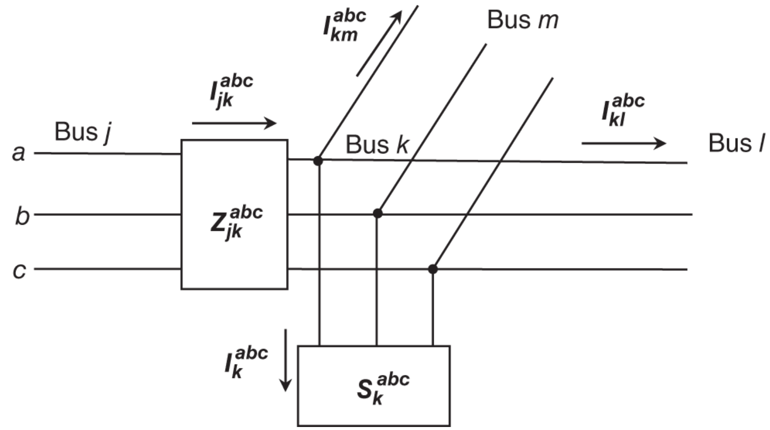


Fig: Feeders splitting at bus k.

- Applying the KCL at bus k gives:

$$I_{jk}^{abc} = I_{kl}^{abc} + I_{km}^{abc} + I_{k}^{abc}$$

- Again, if some of the feeders are two or single phase, the equations are modified to include only the phases that exist for a given line section.

5. Power Flow Studies

5.4 Power Flow Algorithm

- The Source-Load-Iteration (SLI) method is an efficient method for radial distribution system power flow analysis.
- Computation is done in per-unit (pu) for ease of analysis.
- Conversion of the impedances and loads to pu values are required using appropriate base values.

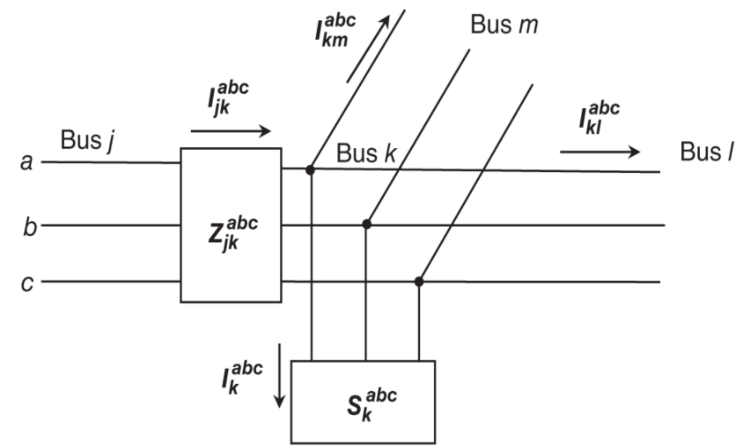


Fig: Feeders splitting at bus k.

5. Power Flow Studies

Step 1:

- The voltage at the substation is fixed or regulated; therefore, it is known. With substation declared as bus 1 of the system, we get

$$\begin{bmatrix} V_1^a \\ V_1^b \\ V_1^c \end{bmatrix} = \begin{bmatrix} 1.0\angle 0^\circ \\ 1.0\angle -120^\circ \\ 1.0\angle 120^\circ \end{bmatrix}$$

5. Power Flow Studies

Step 2:

- Assume a flat profile for all the other bus voltages and set initial values of voltage vectors at all the buses equal to the substation bus, that is

$$V_2^{abc(0)} = V_3^{abc(0)} = \cdots = V_n^{abc(0)} = V_1^{abc}$$

Note: (o) in the superscript is the iteration count, and n is the total number of buses in the system.

5. Power Flow Studies

Step 3:

- Find the current vectors for all the buses that have loads connected to them using:

$$\boldsymbol{I}_i^{\boldsymbol{abc}(\boldsymbol{m})} = \begin{bmatrix} \boldsymbol{I}_i^{\boldsymbol{a}(m)} \\ \boldsymbol{I}_i^{\boldsymbol{b}(m)} \\ \boldsymbol{I}_i^{c(m)} \end{bmatrix}^* = \begin{bmatrix} \boldsymbol{V}_i^{\boldsymbol{a}(m)} & 0 & 0 \\ 0 & \boldsymbol{V}_i^{\boldsymbol{b}(m)} & 0 \\ 0 & 0 & \boldsymbol{V}_i^{\mathrm{c}(m)} \end{bmatrix}^{-1} \begin{bmatrix} \boldsymbol{S}_i^a \\ \boldsymbol{S}_i^b \\ \boldsymbol{S}_i^c \end{bmatrix};$$

$$i = 2,3, \ldots, n$$

Note: m in the superscript is the iteration count, and it should not be confused with bus number m used in the previous section. For DERs, determine P_{i}^{abc} and Q_{i}^{abc} corresponding to the voltage at bus *i* and the selected control mode while considering the reactive power capability.

5. Power Flow Studies

Step 4:

- Find $\boldsymbol{I}_{jk}^{\boldsymbol{a}\mathrm{bc}(m)}$ for each feeder section *j-k* (section between bus *j* and bus *k*) starting from the bus at the edge of the system and sequentially moving toward the source.
- If the system has multiple branches emanating from the main feeder, this must be done for all branches starting from the bus at the edge.

5. Power Flow Studies

Step 5:

- Determine the voltage drop in each feeder section by proceeding from source to load using:

$$\boldsymbol{V}_{ij}^{abc(m)} = \boldsymbol{Z}_{ij}^{abc} \cdot \boldsymbol{I}_{ij}^{abc(m)}$$

- To determine $\boldsymbol{V}_{i}^{abc(m+1)}$, we start from bus 1 (or source) with known voltages and move toward the loads to compute the bus voltages sequentially starting with bus 2 ,

$$\boldsymbol{V}_{2}^{abc(m+1)} = V_{1}^{abc} - V_{12}^{abc(m)}$$

- For all other buses,

$$\boldsymbol{V}_{j}^{abc(m+1)} = \boldsymbol{V}_{i}^{abc(m+1)} - \boldsymbol{V}_{ij}^{abc(m)}, j = 3 \text{ to } n, i = 2 \text{ to } n-1, \text{ and } j < i.$$

5. Power Flow Studies

Step 6:

- Check for the tolerance level, which is the difference between voltages in two successive iterations for every bus and for all three phases at each bus.

$$\Delta \boldsymbol{V}_i^{\boldsymbol{abc}(m+1)} = \left| \boldsymbol{V}_i^{\boldsymbol{abc}(\boldsymbol{m}+\mathbf{1})} - \boldsymbol{V}_i^{\boldsymbol{abc}(\boldsymbol{m})} \right|, i = 2 \text{ to } n$$

- If all the elements of $\Delta \boldsymbol{V}_i^{\boldsymbol{abc}(m+1)}$ are less than ε, which is an arbitrarily selected small number for tolerance, for all the buses, the power flow converges.
- Typically, a value of 0.001 for ε gives good results. If convergence is not achieved, repeat steps 3–6.

6. Voltage Regulation

Maintaining proper voltage in the distribution system is very important. There are numerous methods to improve and control the voltage of primary distribution systems:

1. Use of load tap changing (LTC) transformers.
2. Application of voltage regulators in the distribution substation as well as on the feeders.
3. Application of shunt (or series) capacitors on the feeders or at the distribution substation.
4. Balancing of loads on primary feeders.
5. Increasing feeder conductor size.

6. Voltage Regulation

6. Increasing primary voltage level.
7. Changing feeder sections from single phase to three phase.
8. Transferring of loads from existing feeders to new feeders.
9. Installation of new substations and primary feeders.

While several of these options are usually considered during the planning stages, LTCs, regulators, and capacitors provide the best means of achieving good voltage regulation during the operational stages on a continuous basis.

6. Voltage Regulation

6.1 Voltage Regulation Definition:

- Voltage regulation is the voltage difference between the two ends of a line defined as percentage of the receiving end or downstream voltage.
- Mathematically:

$$\%\text{ Voltage Regulation} = \frac{(V_s - V_r)100}{V_r}$$

where V_s is the magnitude of the sending-end voltage, and V_r is the magnitude of the receiving-end voltage.

6. Voltage Regulation

6.2 Approximate Method for Voltage Regulation:

- From the figure of a feeder on the right,

$$V_s = V_r + I(R + jX)$$

- And the voltage drop on the feeder is,

$$V_s - V_r = I(R + jX)$$

- A phasor diagram for this equation with the condition that current lags voltage by an angle δ is shown in the figure.

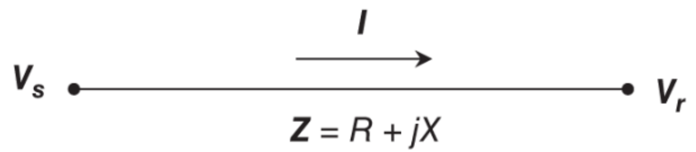


Fig: A feeder of resistance R and reactance X.

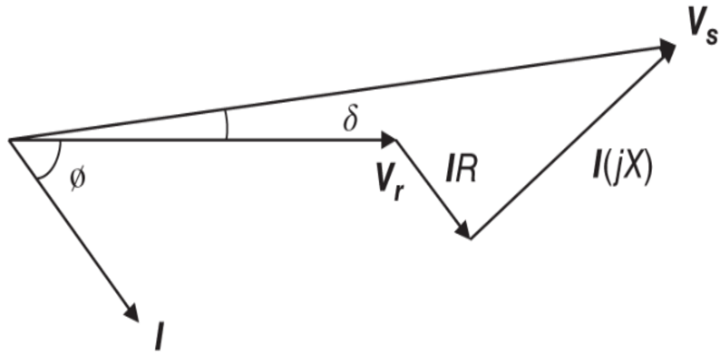


Fig: Phasor Diagram

6. Voltage Regulation

6.2 Approximate Method for Voltage Regulation:

- With $\boldsymbol{V_r}$ as the reference voltage or $\boldsymbol{V_r} = V_r\angle 0°, \boldsymbol{V_S} = V_s\angle\delta°$, and $\boldsymbol{I} = I\angle - \emptyset°$, we get:

$$V_s\cos\delta + jV_s\sin\delta - V_r$$
$$= I(R\cos\emptyset + X\sin\emptyset) + jI(X\cos\emptyset - R\sin\emptyset)$$

- Typically, in distribution systems, R is approximately equal to X, and δ is very small. Hence, we can consider $\cos\delta \approx 1$ and $\sin\delta \approx 0$, which makes the imaginary go to zero, and the real part giving voltage drop per mile becomes:

$$V_s - V_r = I(R\cos\emptyset + X\sin\emptyset)$$

6. Voltage Regulation

- And, the equation of voltage regulation becomes,

$$\%\text{ Voltage Regulation } = \frac{I(R\cos\emptyset + X\sin\emptyset)}{V_r} \times 100$$

- Now, if we consider a load $\boldsymbol{S} = P + jQ$ connected at the receiving end, we can compute the corresponding current

$$\boldsymbol{I} = \left(\frac{P+jQ}{\boldsymbol{V}_r}\right)^*$$

And,

$$I = \frac{S}{V_r}$$

6. Voltage Regulation

which gives

$$\% \text{ Voltage Regulation } = \frac{S(R\cos\emptyset + X\sin\emptyset)}{V_r^2} \times 100$$

$$= \frac{(RP + XQ)}{V_r^2} \times 100$$

- In practice, the approximate voltage drop is an acceptable measure since the error between the exact and approximate values is negligible.

6. Voltage Regulation

- In distribution systems, usually Vs, the voltage at the substation, is controlled and held constant for varying loading conditions, which implies that the voltage drop and hence Vr at other buses changes.
- Obtaining Vr at a given bus for the given impedance and load values requires iterative solution using power flow method.

6. Voltage Regulation

Example:

Consider a 12.47-kV feeder with three point loads as shown in Figure 4.13. The loads are

S2 = 2.5 MVA with 0.92 lagging power factor,

S3 = 3.0 MVA with 0.90 lagging power factor, and

S4 = 2.0 MVA with 0.95 lagging power factor.

The given impedance of the feeder, z, is (0.258 + j 0.6644) Ω/mi or 0.7127∠68.78° Ω/mi. Find the percent voltage drop at bus 4 of the primary feeder for the stipulated load conditions using the approximate method.

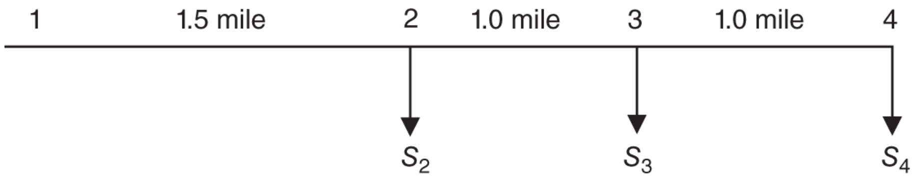


Fig: A single-phase feeder supplying three point loads.

6. Voltage Regulation

Solution:

- First, compute impedances, Z, of the line sections by multiplying z by l, which is the length of the feeder sections, or:

$$Z_{12} = z \cdot l_{12} = (0.7127\angle 68.78°\ \Omega/\text{mi})(1.5\ \text{mi}) = 1.0691\angle 68.78°\ \Omega$$
$$= 0.3870 + j0.9966\ \Omega$$
$$Z_{23} = z \cdot l_{23} = (0.7127\angle 68.78°\ \Omega/\text{mi})(1\ \text{mi}) = 0.7127\angle 68.78°\ \Omega$$
$$= 0.258 + j0.6644\ \Omega$$
$$Z_{34} = z \cdot l_{34} = (0.7127\angle 68.78°\ \Omega/\text{mi})(1\ \text{mi}) = 0.7127\angle 68.78°\ \Omega$$
$$= 0.258 + j0.6644\ \Omega$$

- Next, compute the line-to-neutral voltage at bus 1 for the given line-to-line voltage for the feeder, or:

$$V_1 = \frac{12470}{\sqrt{3}} = 7200V$$

6. Voltage Regulation

Assume this voltage to be the reference voltage, or V1 = 7200∠0°. We also assume that the voltage at each bus remains at 7200∠0°, and we compute the load currents for each bus, or:

$$I_4 = (S_4 / V_4)^* = (2.0\times10^6 / 7200) \angle -\cos^{-1}(0.95)\ \mathrm{A}$$
$$= 92.60 \angle -18.19°\ \mathrm{A}$$
$$I_3 = (S_3 / V_3)^* = (3.0\times10^6 / 7200) \angle -\cos^{-1}(0.92)\ \mathrm{A}$$
$$= 138.89 \angle -25.84°\ \mathrm{A}$$
$$I_2 = (S_2 / V_2)^* = (2.5\times10^6 / 7200) \angle -\cos^{-1}(0.92)\ \mathrm{A}$$
$$= 115.74 \angle -23.07°\ \mathrm{A}$$

- Now, compute the currents in the feeder sections using KCL:

$$I_{34} = I_4 = 92.60 \angle -18.19°\ \mathrm{A}$$
$$I_{23} = I_{34} + I_3 = 92.60 \angle -18.19°\ \mathrm{A} + 138.89 \angle -25.84°\ \mathrm{A}$$
$$= 230.9950 \angle -22.78°\ \mathrm{A}$$
$$I_{12} = I_{23} + I_2 = 230.9950 \angle -22.78°\ \mathrm{A} + 115.74 \angle -23.07°\ \mathrm{A}$$
$$= 346.73 \angle -22.78°\ \mathrm{A}$$

6. Voltage Regulation

- Compute voltage drops in each feeder section:

$$\mathrm{VD}_{12} = \mathrm{I}_{12}\,[\,\mathrm{R}_{12}\cos(22.88°) + \mathrm{X}_{12}\sin(22.88°)\,]$$

$$= 346.7340\,[\,0.3870 \times 0.9213 + 0.6644 \times 0.3888\,] = 257.97\ \mathrm{V}$$

$$\mathrm{VD}_{23} = \mathrm{I}_{23}\,[\,\mathrm{R}_{23}\cos(22.78°) + \mathrm{X}_{23}\sin(22.78°)\,]$$

$$= 230.9950\,[\,0.2580 \times 0.9219 + 0.6644 \times 0.3872\,] = 114.37\ \mathrm{V}$$

$$\mathrm{VD}_{34} = \mathrm{I}_{34}\,[\,\mathrm{R}_{34}\cos(18.90°) + \mathrm{X}_{34}\sin(18.19°)\,]$$

$$= 92.60\,[\,0.2580 \times 0.95 + 0.6644 \times 0.3122\,] = 41.90\ \mathrm{V}$$

Thus, the total voltage drop in the entire feeder is the sum of the voltage drops in individual sections, or:

$$\mathrm{VD}_{14} = \mathrm{VD}_{34} + \mathrm{VD}_{23} + \mathrm{VD}_{12}$$

$$= 41.90 + 114.37 + 257.97 = 414.24\ \mathrm{V}$$

$$\mathrm{V}_{\mathrm{r}} = 7200 - 414.24 = 6815.76\ \mathrm{V}$$

6. Voltage Regulation

- Therefore, the percent voltage regulation is

$$\%V\text{oltage Regulation} = \frac{414.24}{6815.74} \times 100 = 6.07\%$$

- Verify the accuracy on this result by finding the exact solution for this problem by determining voltages with power flow using the source-to-load iterative method. You will find that the error is minimal in this case due to the power factors of all three loads being close to each other.

6. Voltage Regulation

6.3 Voltage Drop on Radial Feeders with Uniformly Distributed Load

- In the previous section, we considered spot loads connected at the buses.
- In this section, we consider a case where the loads are distributed uniformly across the length of the feeder in a rectangular service area.
- Also, we generalize it for a three-phase feeder, assuming that all the three phases have identical values of resistance and reactance.

6. Voltage Regulation

6.3 Voltage Drop on Radial Feeders with Uniformly Distributed Load

- Consider the current density to be k A/mile for the feeder shown in the Figure. With l as the length of the feeder, the current at the sending end is $I_s = kl$, and the current at the receiving end is 0.

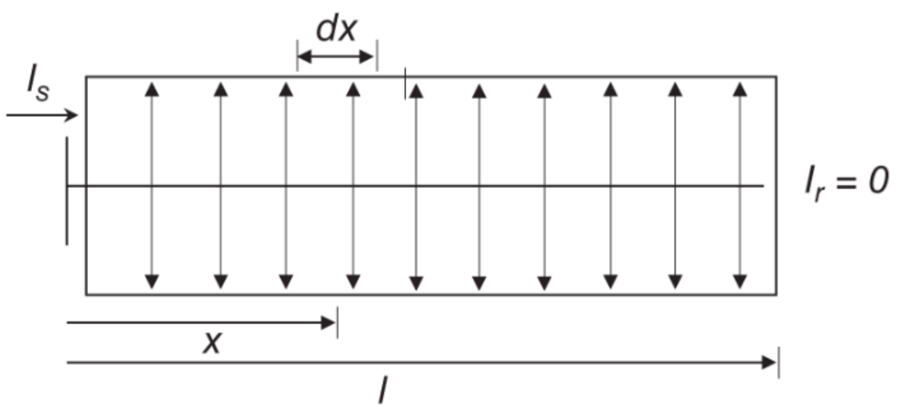


Fig: Feeder with load distributed uniformly in a rectangular service area

6. Voltage Regulation

- Fig (b) shows the plot of the magnitude of a current assuming that the current throughout the feeder has the same phase angle.
- From this figure, we get

$$I_x = I_s - kx$$

or,

$$I_x = I_s - \frac{I_s}{l}x$$

$$I_x = I_s\left(1 - \frac{x}{l}\right)$$

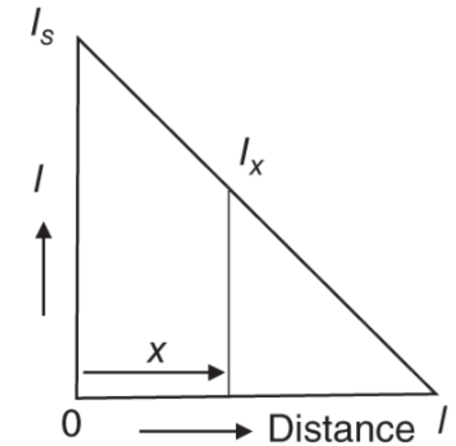


Fig (b): Current on the feeder

- Consider a differential element of the feeder of length dx at a distance x. With z ohms/mile as impedance of the feeder, the differential voltage across dx is,

$$dV_D = I_x z dx = I_s\left(1 - \frac{x}{l}\right) z dx$$

6. Voltage Regulation

- Therefore, the voltage drop on the feeder is

$$V_{Dl} = \int_0^l I_s\left(1 - \frac{x}{l}\right) z dx = \frac{1}{2} I_s z l V$$

- We can also compute losses on the feeder by considering differential power loss with r ohms/mile as the resistance of the feeder.

$$dP_{LSx} = I_x^2 r dx = I_s^2 \left(1 - \frac{x}{l}\right)^2 r dx$$

Then, the power loss per phase is

$$P_{LS} = \int_0^l I_s^2 \left(1 - \frac{x}{l}\right)^2 r dx = \frac{1}{3} I_s^2 r l$$

6. Voltage Regulation

- The total power loss for the three phases is

$$P_{LS3\emptyset} = I_s^2 rl$$

- The results shown above are useful for planning studies because while planning we do not have the system topology and actual load values available. All we have are projected values.

- These approximations are also useful in optimizing operating scenarios where precise calculations may slow down the process of obtaining the final solution. In those cases, approximation can be used to narrow the solution space, and precise calculations can be implemented on a smaller solution space, thus, speeding the overall computation in search for the optimal solution.

6. Voltage Regulation

6.4 Voltage Drop on Radial Feeders Serving a Triangular Area

- This section, we consider a case in which the service area of a feeder is triangular with load distributed uniformly in the service area.
- Let a be the current density in amperes per square mile.

$$a = \frac{2I_s}{lh}$$

- Next, we get an expression for current I_X at distance x.

$$I_x = I_s - a\frac{h'x}{2}$$

Substituting the value of a gives

$$I_s - I_s\frac{2}{lh}\frac{h'x}{2}$$

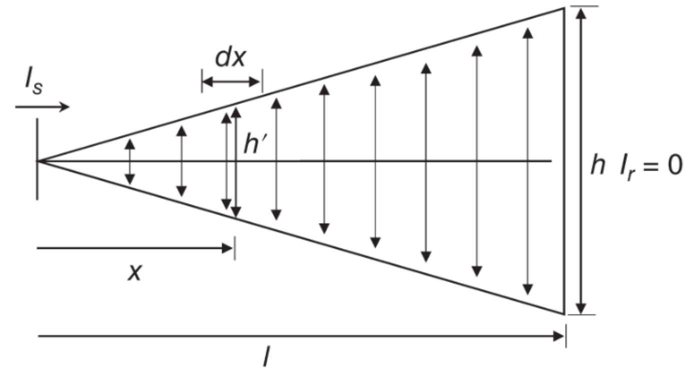


Fig: Feeder serving a triangular area with fixed load density

6. Voltage Regulation

Note that $\frac{h'}{h} = \frac{x}{l}$. Substituting this ,

$$I_x = I_s - I_s \frac{x^2}{l^2}$$

- Again, consider a differential element dx and write an expression for the differential voltage drop

$$dV_D = I_x z dx$$

- Further, the voltage drop across the feeder is

$$V_{Dl} = \int_0^l I_s \left(1 - \frac{x^2}{l^2}\right) z dx$$

or

$$V_{Dl} = \frac{2}{3} I_s z l$$

6. Voltage Regulation

Similarly, we can find power loss, which is

$$dP_{LSx} = I_x^2 r dx = I_S^2 \left(1 - \frac{x^2}{l^2}\right)^2 r dx$$

Then, the power loss per phase is

$$P_{LS} \quad = \int_0^l I_S^2 \left(1 - \frac{x^2}{l^2}\right)^2 r dx = \frac{8}{15} I_S^2 r l$$

And,

$$P_{LS3\emptyset} = \frac{8}{5} I_S^2 r l$$

7. Fault Calculation

- The standard procedure for fault calculation in power systems requires determining Thevenin's equivalent for positive-, negative-, and zero-sequence networks at the point of fault.
- The sequence networks are connected to each other based on the type of fault.
- However, decoupling of circuits in the sequence domain works only under the conditions of symmetry.

7. Fault Calculation

- For example, consider a three-phase feeder with impedance matrix in the phase domain as shown below,

$$\begin{bmatrix} Z_{aa} & Z_{ab} & Z_{ac} \\ Z_{ba} & Z_{bb} & Z_{bc} \\ Z_{ca} & Z_{cb} & Z_{cc} \end{bmatrix}$$

- The conditions of symmetry require that,

$$Z_{aa} = Z_{bb} = Z_{cc}, \text{ and } Z_{ab} = Z_{ac} = Z_{bc}$$

7. Fault Calculation

- While for transmission lines these are satisfied with transposition, they are not feasible for distribution feeders. Distribution feeders are rarely transposed and also can be of two or one phase in addition to three phases.

- Since single- and two-phase feeders do not have models in the sequence domain, symmetrical component-based analysis is not applicable to them. Also, for three-phase part of the system, symmetrical components do not provide any advantage because the sequence impedance matrix is not diagonal.

7. Fault Calculation

- Hence, positive-, negative-, and zero-sequence impedances cannot be decoupled from one another.
- However, we can implement a solution technique in phase domain to determine fault currents in distribution systems. We initially consider a radial distribution system connected to the bulk power system with no additional sources in the system.

7.1 Prefault System

- A general distribution feeder connected to the substation bus with multiple feeder sections as shown in the Figure. While the main feeder is a three-phase feeder, the laterals can be three, two, or single phase. Each feeder section is modeled by its impedance matrix.
- For illustration, we are considering delta-wye-grounded connection for the transformer, but the method will work for other configurations too.

7.1 Prefault System

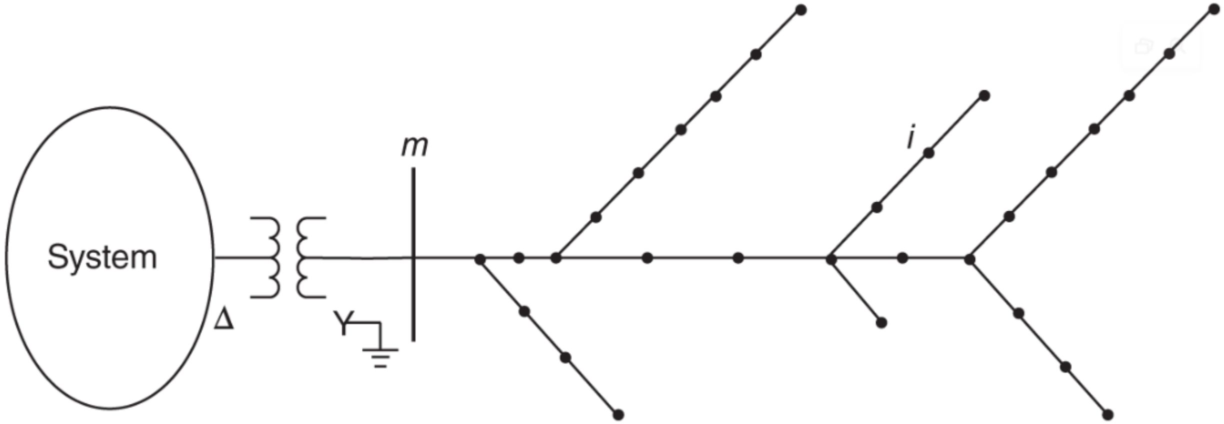


Fig: An example Distribution System

- The system in the figure represented in the oval represents the entire power system upstream of the transformer.
- This system can be represented by a Thevenin equivalent impedance matrix and a voltage source vector for the three phases.

7.1 Prefault System

- The equivalent voltage source V_s is considered balanced with 1 pu magnitude, i.e.

$$\begin{bmatrix} \boldsymbol{V}_s^a \\ \boldsymbol{V}_s^b \\ \boldsymbol{V}_s^c \end{bmatrix} = \begin{bmatrix} 1.0\angle 0^\circ \\ 1.0\angle -120^\circ \\ 1.0\angle 120^\circ \end{bmatrix}$$

where subscript *s* is the source.

- All the prefault load currents are considered to be 0 with the assumption that load currents are much smaller compared to the fault current.

7.1 Prefault System

- Directly determining the Thevenin equivalent matrix in the phase domain is difficult. An indirect approach is considered due to the difficulty.
- Utilities usually have information on equivalent positive-, negative-, and zero-sequence impedance values at different buses in the transmission system.
- The values can also be computed using the approach discussed in Section 2 of the slide (Modeling of Source Impedance).
- The values are denoted as Z_1^{sm}, Z_2^{sm}, and Z_0^{sm} at bus m.
- The zero-sequence impedance value only includes the impedance of the transformer due to the selected transformer connection.

7.1 Prefault System

- Delta/wye-grounded transformers create an open circuit in the zero-sequence circuit, where current flows through the transformer only on the wye-grounded side.
- The transmission system is assumed to be fully balanced. As a result, the sequence networks are decoupled.

7.1 Prefault System

- The impedance matrix of the system in the sequence domain can be written as follows:

$$\begin{bmatrix} \boldsymbol{Z}_0^{sm} & 0 & 0 \\ 0 & \boldsymbol{Z}_1^{sm} & 0 \\ 0 & 0 & \boldsymbol{Z}_2^{sm} \end{bmatrix}$$

- Now, we use transformation to convert this matrix to phase domain

$$\begin{bmatrix} Z_{aa}^{sm} & Z_{ab}^{sm} & Z_{ac}^{sm} \\ Z_{ba}^{sm} & Z_{bb}^{sm} & Z_{bc}^{sm} \\ Z_{ca}^{sm} & Z_{cb}^{sm} & Z_{cc}^{sm} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \boldsymbol{Z}_0^{sm} & 0 & 0 \\ 0 & Z_1^{sm} & 0 \\ 0 & 0 & \boldsymbol{Z}_2^{sm} \end{bmatrix} \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}$$

7.1 Prefault System

where **a** is a complex number equal to $1\angle 120^{o}$

- Let bus *i* be the candidate bus for faults.
- Since all the feeder sections between the substation bus (bus *m*) and the faulted bus are in series, we can add their impedance matrices to compute the equivalent impedance matrix.
- All the feeder sections not in the path from the substation to the faulted bus do not influence the fault current and thus are not included in the calculations.
- Consider this matrix to be: $\begin{bmatrix} Z_{aa}^{mi} & Z_{ab}^{mi} & Z_{ac}^{mi} \\ Z_{ba}^{mi} & Z_{bb}^{mi} & Z_{bc}^{mi} \\ Z_{ca}^{mi} & Z_{cb}^{mi} & Z_{cc}^{mi} \end{bmatrix}$

7.1 Prefault System

- Now, this matrix is added to the system impedance matrix to get the impedance matrix from the equivalent source to the faulted bus,

$$\begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} = \begin{bmatrix} Z_{aa}^{sm} & Z_{ab}^{sm} & Z_{ac}^{sm} \\ Z_{ba}^{sm} & Z_{bb}^{sm} & Z_{bc}^{sm} \\ Z_{ca}^{sm} & Z_{cb}^{sm} & Z_{cc}^{sm} \end{bmatrix} + \begin{bmatrix} Z_{aa}^{mi} & Z_{ab}^{mi} & Z_{ac}^{mi} \\ Z_{ba}^{mi} & Z_{bb}^{mi} & Z_{bc}^{mi} \\ Z_{ca}^{mi} & Z_{cb}^{mi} & Z_{cc}^{mi} \end{bmatrix}$$

Note: This summation can only be done if both matrices on the right-hand side are in per unit, or impedances are referred to the low-voltage side of the transformer due to different voltage levels of the distribution system and the bulk power system.

7.2 Three – Phase Fault

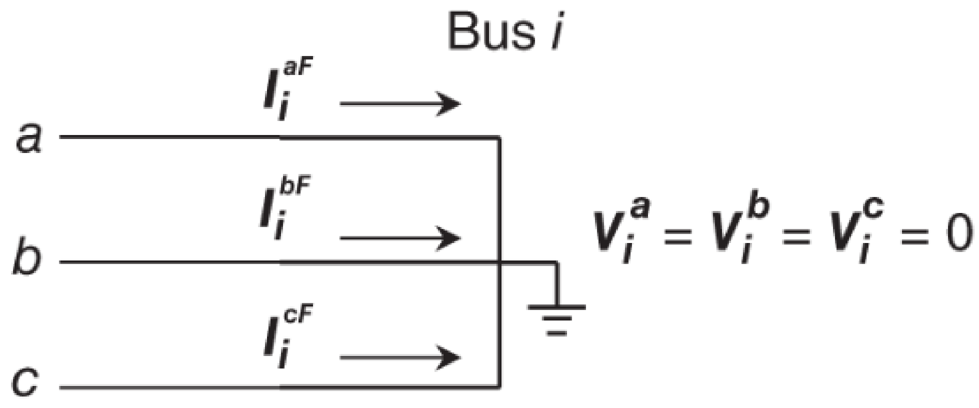


Fig: Three – phase fault at bus *i*

- For a three-phase fault at bus *i*, current will flow from the substation to bus *i*.
- Since there is no additional source, no other currents will flow.
- Also, we assume that the fault impedance is 0.

7.2 Three – Phase Fault

- If the fault is a high impedance fault, these assumptions will not be fully valid. The Figure above shows the conditions for a three-phase fault.

- The fault currents are, $I_i^{abcF} = \begin{bmatrix} I_i^{aF} \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix}$.

7.2 Three – Phase Fault

- The equation from the voltage drop from the source to bus *i* is,

$$\begin{bmatrix} V_s^a - V_i^a \\ V_s^b - V_i^b \\ V_s^c - V_i^c \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} \begin{bmatrix} I_i^{aF} \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix}$$

- For a solidly grounded fault, all the voltages at bus *i* will be 0.

7.2 Three – Phase Fault

- Applying this condition and substituting values of the source bus voltage, we get,

$$\begin{bmatrix} 1.0\angle 0^\circ - 0 \\ 1.0\angle -120^\circ - 0 \\ 1.0\angle 120^\circ - 0 \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} \begin{bmatrix} I_i^{aF} \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix}$$

- In the next step, we get,

$$\begin{bmatrix} I_i^{aF} \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix}^{-1} \begin{bmatrix} 1.0\angle 0^\circ \\ 1.0\angle -120^\circ \\ 1.0\angle 120^\circ \end{bmatrix}$$

7.2 Three – Phase Fault

- If the system has additional sources, they can be applied one at a time while removing the equivalent source at the substation.
- Source impedance matrix needs to be known and the equivalent impedance matrix from each source to bus *i* must be determined.
- Voltages at the source terminal are required for the calculations.

7.2 Three – Phase Fault

- For inverter-based sources, their characteristics under faults and operating rules must be known.
- Some inverter-based sources are automatically disconnected during fault, and most are adjusted to produce a smaller current by adjusting the impedance of inverter, which makes fault calculations challenging.
- However, if we can compute fault currents for all the sources one at a time, all the fault currents can be added using superposition to compute the cumulative current.
- Since the circuit is linear, superposition can be applied without affecting the results.

7.3 Double-Line-to-Ground (DLG) Fault

- For a fault at bus i, current will flow from the substation to bus i on the two faulted phases and no current on the third phase. As shown in the Figure, a fault between phases b and c is considered.

- The currents for this fault are

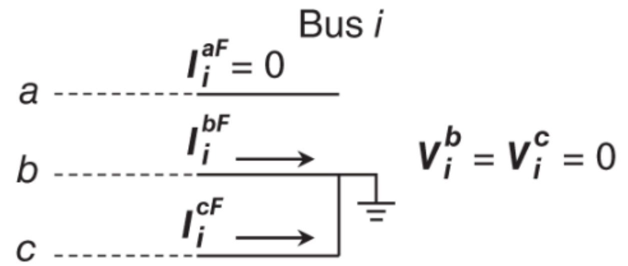


Fig: DLG fault between phases *b* and *c* at bus *i*.

$$I_i^{abcF} = \begin{bmatrix} 0 \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix}$$

- The voltage drop equation from the source to bus i is:

$$\begin{bmatrix} 1.0\angle 0^\circ - V_i^a \\ 1.0\angle -120^\circ - 0 \\ 1.0\angle 120^\circ - 0 \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{i} \\ Z_{ca}^{si} & Z_{cb}^{i} & Z_{cc}^{s} \end{bmatrix} \begin{bmatrix} 0 \\ I_i^{bF} \\ I_i^{cF} \end{bmatrix}$$

7.3 Double-Line-to-Ground (DLG) Fault

- Discard the first row and the first column and write the equations in a reduced form, as,

$$\begin{bmatrix} 1.0\angle-120° \\ 1.0\angle 120° \end{bmatrix} = \begin{bmatrix} Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} \begin{bmatrix} I_i^{bF} \\ I_i^{cF} \end{bmatrix}$$

- Further,

$$\begin{bmatrix} \boldsymbol{I}_i^{bF} \\ \boldsymbol{I}_i^{cF} \end{bmatrix} = \begin{bmatrix} \boldsymbol{Z}_{bb}^{si} & \boldsymbol{Z}_{bc}^{si} \\ \boldsymbol{Z}_{cb}^{si} & \boldsymbol{Z}_{cc}^{si} \end{bmatrix}^{-1} \begin{bmatrix} 1.0\angle-120° \\ 1.0\angle 120° \end{bmatrix}$$

- The same approach can be used for faults on any other two phases.

7.4 Single-Line-to-Ground (SLG) Fault

- For a fault at bus *i*, current will flow from the substation to bus *i* on the faulted phase and no current on the other two phases.
- For illustration, we consider a fault on phase *a* as shown in the figure.
- The currents for this fault are

$$I_i^{abcF} = \begin{bmatrix} I_i^{aF} \\ 0 \\ 0 \end{bmatrix}$$

- We can write an equation for voltage drop from the equivalent source to bus i as,

$$\begin{bmatrix} 1.0\angle 0^\circ - 0 \\ 1.0\angle -120^\circ - V_i^b \\ 1.0\angle 120^\circ - V_i^c \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} \begin{bmatrix} I_i^{aF} \\ 0 \\ 0 \end{bmatrix}$$

7.4 Single-Line-to-Ground (SLG) Fault

- Discard the last two equations and keep only the first one, or

$$1.0\angle 0^\circ = Z_{aa}^{si} I_i^{aF}$$

Therefore,

$$I_i^{aF} = \frac{1.0\angle 0^\circ}{Z_{aa}^{si}}$$

- Again, the same approach can be used for faults on other phases.

7.5 Line-to-Line (LL) Fault

- For an LL fault at bus *i*, current will flow from the substation to bus *i* on the faulted phases and no current on the third phases.
- For illustration, we consider a fault between phase *b* and *c* as shown in the figure.
- For this fault, the currents will not flow to the ground but return through the second phase, as,

$$I_i^{cF} = -I_i^{bF}$$

- Hence, the currents for this fault are:

$$I_i^{abcF} = \begin{bmatrix} 0 \\ I_i^{bF} \\ -I_i^{bF} \end{bmatrix}$$

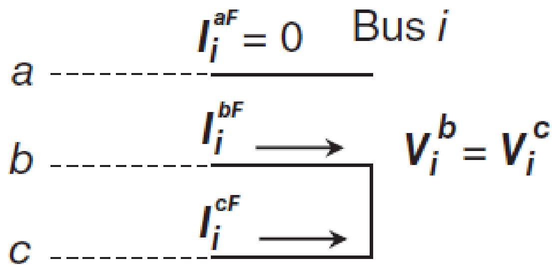


Fig: DLG fault between phases *b* and *c* at bus *i*.

7.5 Line-to-Line (LL) Fault

- Also, the voltages at bus i for the faulted phases are equal,

$$V_i^b = V_i^c$$

- Therefore, the equation for voltage drop from the equivalent source to bus i is

$$\begin{bmatrix} 1.0\angle 0^\circ - V_i^a \\ 1.0\angle -120^\circ - V_i^b \\ 1.0\angle 120^\circ - V_i^b \end{bmatrix} = \begin{bmatrix} Z_{aa}^{si} & Z_{ab}^{si} & Z_{ac}^{si} \\ Z_{ba}^{si} & Z_{bb}^{si} & Z_{bc}^{si} \\ Z_{ca}^{si} & Z_{cb}^{si} & Z_{cc}^{si} \end{bmatrix} \begin{bmatrix} 0 \\ I_i^{bF} \\ -I_i^{bF} \end{bmatrix}$$

- Expanding the last two equations gives

$$1.0\angle -120^\circ - V_i^b = Z_{bb}^{si} I_i^{bF} - Z_{bc}^{si} I_i^{bF} = \left(Z_{bb}^{si} - Z_{bc}^{si}\right) I_i^{bF}$$

And.

$$1.0\angle 120^\circ - V_i^b = Z_{cb}^{si} I_i^{bF} - Z_{cc}^{si} I_i^{bF} = \left(Z_{cb}^{si} - Z_{cc}^{si}\right) I_i^{bF}$$

7.5 Line-to-Line (LL) Fault

- Rearranging and expressing in the matrix form gives

$$\begin{bmatrix} I_i^{bF} \\ V_i^{b} \end{bmatrix} = \begin{bmatrix} \left(Z_{bb}^{si} - Z_{bc}^{si}\right) & 1 \\ \left(Z_{cb}^{si} - Z_{cc}^{si}\right) & 1 \end{bmatrix}^{-1} \begin{bmatrix} 1.0\angle -120° \\ 1.0\angle 120° \end{bmatrix}$$

- Solving it gives both $\boldsymbol{I_i^{bF}}$ and $\boldsymbol{V_i^{b}}$ as well as $\boldsymbol{I_i^{cF}}$ and $\boldsymbol{V_i^{c}}$.

7.6 Symmetrical Component-based Fault Analysis

- Although symmetrical component-based fault analysis does not work well for distribution systems, it can be used to get approximate results only for the part of the system that has all the three phases.
- The first step is to determine the three-phase impedance matrix from the substation bus (bus *m*) to the faulted bus (bus *i*).
- The general equation for this impedance matrix is given by,

$$\begin{bmatrix} Z_{aa}^{mi} & Z_{ab}^{mi} & Z_{ac}^{mi} \\ Z_{ba}^{mi} & Z_{bb}^{mi} & Z_{bc}^{mi} \\ Z_{ca}^{mi} & Z_{cb}^{mi} & Z_{cc}^{mi} \end{bmatrix}$$

7.6 Symmetrical Component-based Fault Analysis

- The next step is to transform this matrix to sequence domain, which gives:

$$\begin{bmatrix} Z_{00}^{mi} & Z_{01}^{mi} & Z_{02}^{mi} \\ Z_{10}^{mi} & Z_{11}^{mi} & Z_{12}^{mi} \\ Z_{20}^{mi} & Z_{21}^{mi} & Z_{22}^{mi} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} Z_{aa}^{mi} & Z_{ab}^{mi} & Z_{ac}^{mi} \\ Z_{ba}^{mi} & Z_{bb}^{mi} & Z_{bc}^{mi} \\ Z_{ca}^{mi} & Z_{cb}^{mi} & Z_{cc}^{mi} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix}$$

7.6 Symmetrical Component-based Fault Analysis

- The resulting sequence impedance matrix is not diagonal, but for approximation, we discard the off-diagonal terms.
- The diagonal entries are considered as the zero-, positive, and negative-sequence impedances of the distribution system from the substation bus to the point of fault.
- To determine the pre-fault circuits in the sequence domain, we add the respective impedances from the equivalent source to the substation bus and the impedances from the substation bus to the faulted bus.

7.6 Symmetrical Component-based Fault Analysis

- All impedances must be converted to per unit before adding them.
- The voltage source in the phase domain is balanced, providing a source only for the positive sequence.
- Hence, we can determine the three pre-fault sequence domain circuits as shown in the figure (next slide).

7.6 Symmetrical Component-based Fault Analysis

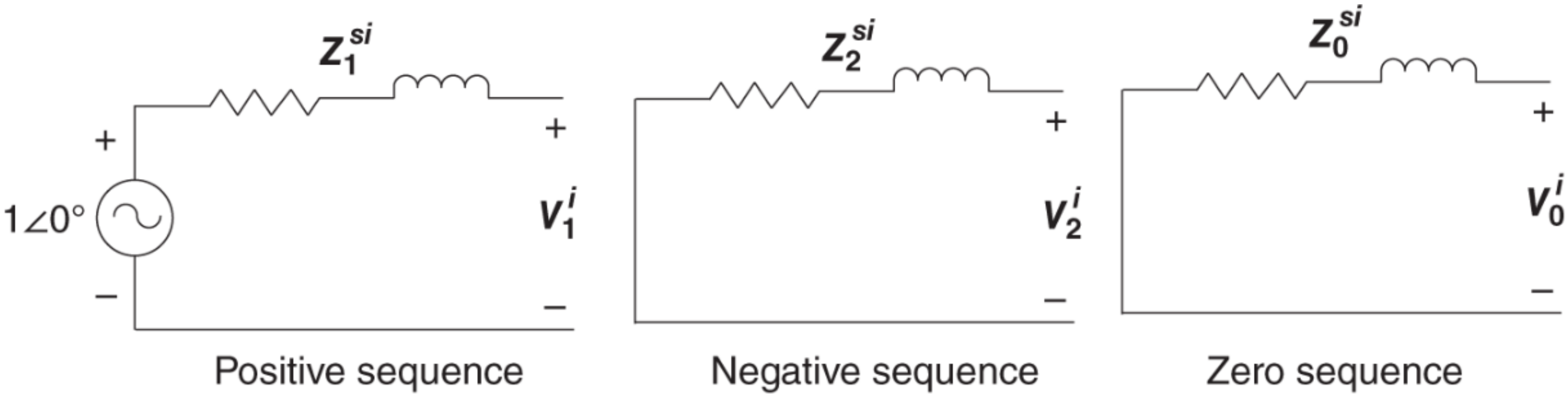


Fig: Pre-fault positive-, negative-, and zero-sequence equivalent circuits with respect to bus *i*.

- Note that in the figure,

$$Z_1^{si} = Z_1^{sm} + Z_{11}^{mi}$$
$$Z_2^{si} = Z_2^{sm} + Z_{22}^{mi}$$
$$Z_0^{si} = Z_0^{sm} + Z_{00}^{mi}$$

7.6.1 Three-phase Fault

- For a three-phase fault, the system stays balanced, and the negative- and the zero-sequence currents are 0.
- To compute the positive-sequence current, we create a short circuit across the positive-sequence circuit as shown in Figure.

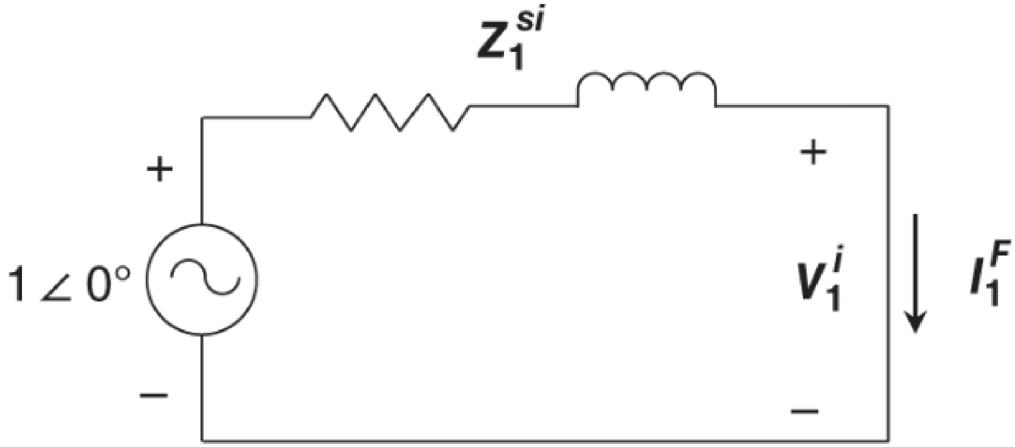


Fig: Three-phase fault in sequence domain.

7.6.1 Three-phase Fault

- From this circuit,

$$I_1^F = \frac{1.0\angle 0^\circ}{Z_1^{si}}$$

- Now, we convert them to the phase domain,

$$\begin{bmatrix} \boldsymbol{I}_i^{aF} \\ \boldsymbol{I}_i^{bF} \\ \boldsymbol{I}_i^{cF} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a}^2 & \boldsymbol{a} \\ 1 & \boldsymbol{a} & \boldsymbol{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ \boldsymbol{I}_1^F \\ 0 \end{bmatrix} = \begin{bmatrix} \boldsymbol{I}_1^F \\ \boldsymbol{I}_1^F \angle -120^\circ \\ \boldsymbol{I}_1^F \angle 120^\circ \end{bmatrix}$$

7.6.2 DLG Fault

The conditions for this fault in phase domain are

$$\boldsymbol{I}_i^{aF} = 0 \text{ and } \boldsymbol{V}_i^{b} = \boldsymbol{V}_i^{c}.$$

Transforming these conditions to sequence domain gives,

$$I_0^F + I_1^F + I_2^F = 0$$

And,

$$V_0^F + aV_1^F + a^2V_2^F = V_0^F + a^2V_1^F + aV_2^F$$

Simplifying gives,

$$(a - a^2)V_1^F = (a - a^2)V_2^F$$

or,

$$V_1^F = V_2^F$$

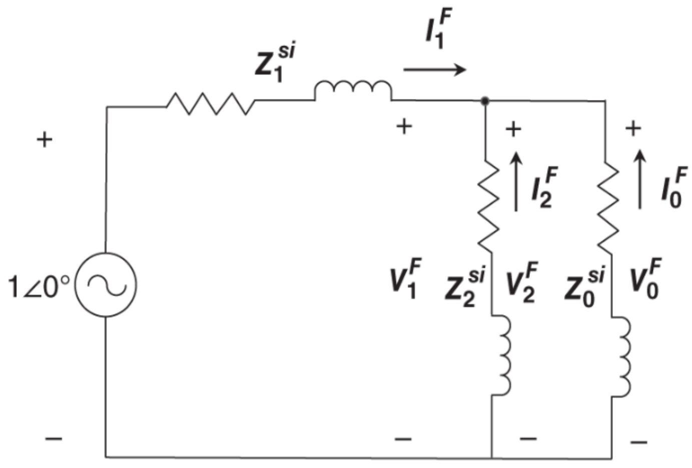


Fig: DLG fault in sequence domain.

7.6.2 DLG Fault

Also,

$$V_i^b = V_0^F + aV_1^F + a^2V_2^F = 0$$

or,

$$V_0^F + (a + a^2)V_1^F = V_0^F - V_1^F = 0$$

Therefore,

$$V_0^F = V_1^F = V_2^F$$

- We use conditions given by:

$$Z_1^{si} = Z_1^{sm} + Z_{11}^{mi}$$
$$Z_2^{si} = Z_2^{sm} + Z_{22}^{mi}$$
$$Z_0^{si} = Z_0^{sm} + Z_{00}^{mi}$$

and $V_0^F = V_1^F = V_2^F$, to create the circuit in sequence domain shown in the Figure .

$$I_1^F = \frac{1.0\angle 0^\circ}{Z_1^{si} + \left(Z_2^{si} \| Z_0^{si}\right)} = \frac{1.0\angle 0^\circ}{Z_1^{si} + \frac{Z_2^{si} + Z_0^{si}}{Z_2^{si} Z_0^{si}}}$$

7.6.2 DLG Fault

$$I_2^F = -I_1^F\left(\frac{Z_0^{si}}{Z_2^{si}+Z_0^{si}}\right)$$

and,

$$I_0^F = -I_1^F\left(\frac{Z_2^{si}}{Z_2^{si}+Z_0^{si}}\right)$$

The sequence currents and voltages can be transformed to the phase domain using the relevant equations.

7.6.3 SLG Fault

- The conditions for this fault in phase domai are $\boldsymbol{I_i^{bF}} = \boldsymbol{I_i^{cF}} = 0$ and $\boldsymbol{V_i^{a}} = 0$.
- Transforming these conditions to sequenc domain gives

$$\begin{bmatrix} I_0^F \\ I_1^F \\ I_2^F \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} I_i^{aF} \\ 0 \\ 0 \end{bmatrix} = \frac{1}{3}\begin{bmatrix} I_i^{aF} \\ I_i^{aF} \\ I_i^{aF} \end{bmatrix}$$

or,

$$I_0^F = I_1^F = I_2^F = \frac{1}{3} I_i^{aF}$$

Also,

$$V_i^a = V_0^F + V_1^F + V_2^F = 0$$

- Based on the above two equations, we can create a circuit shown in the Figure.

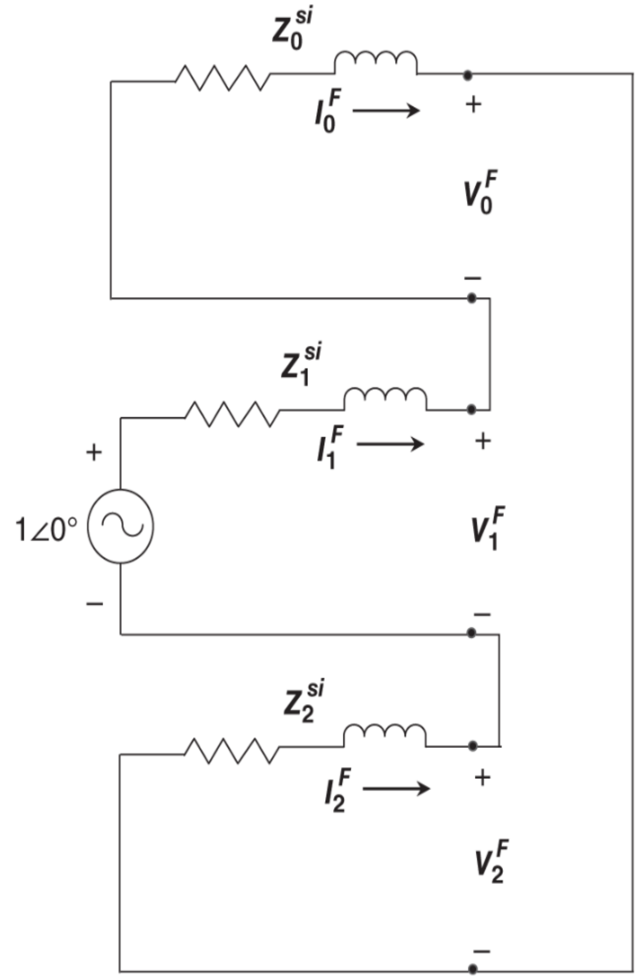


Fig: SLG fault in sequence domain.

7.6.3 SLG Fault

- From this circuit, we get

$$I_0^F = I_1^F = I_2^F = \frac{1.0\angle 0^\circ}{Z_0^{si} + Z_1^{si} + Z_2^{si}}$$

And,

$$I_i^{aF} = 3I_1^F$$

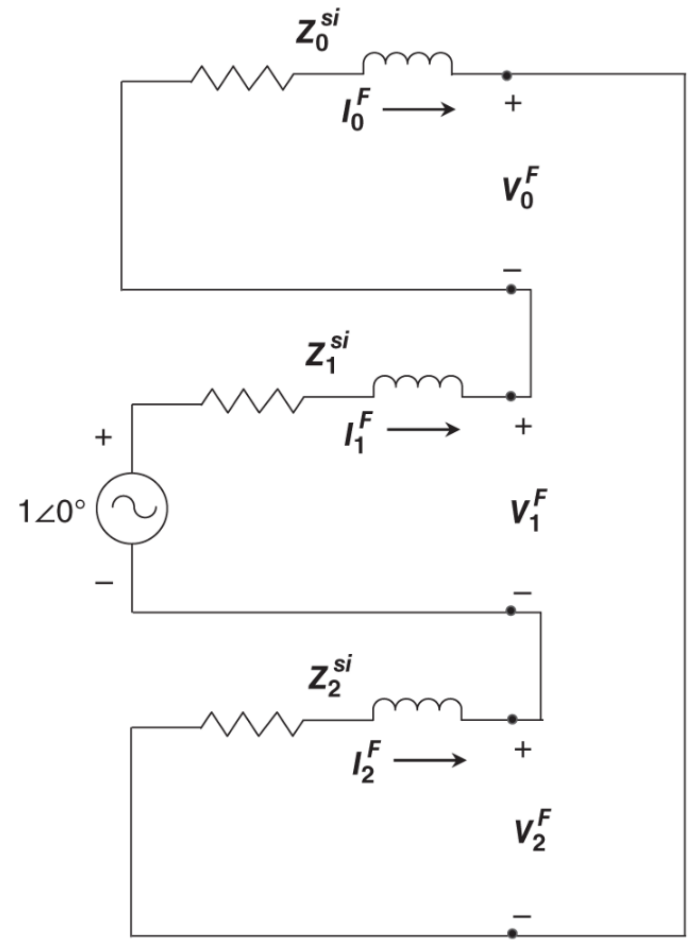


Fig: SLG fault in sequence domain.

7.6.4 LL Fault

- The conditions for this fault in the phase domain are $\boldsymbol{I}_i^{aF} = 0, \boldsymbol{I}_i^{cF} = -\boldsymbol{I}_{\boldsymbol{i}}^{\boldsymbol{bF}}$, and $\boldsymbol{V}_{\boldsymbol{i}}^{b} = \boldsymbol{V}_{\boldsymbol{i}}^{\boldsymbol{c}}$.
- Transforming these conditions to sequence domain gives

$$\begin{bmatrix} I_0^F \\ \boldsymbol{I}_1^F \\ \boldsymbol{I}_2^F \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \boldsymbol{a} & \boldsymbol{a}^2 \\ 1 & \boldsymbol{a}^2 & \boldsymbol{a} \end{bmatrix}\begin{bmatrix} 0 \\ \boldsymbol{I}_i^{bF} \\ -\boldsymbol{I}_i^{bF} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 0 \\ (\boldsymbol{a} - \boldsymbol{a}^2)\boldsymbol{I}_i^{bF} \\ (\boldsymbol{a}^2 - \boldsymbol{a})\boldsymbol{I}_i^{bF} \end{bmatrix}$$

and

$$V_0^F + a^2V_1^F + aV_2^F = V_0^F + aV_1^F + a^2V_2^F$$

Therefore,

$$I_2^F = -I_1^F = \frac{1}{3}(a - a^2)I_i^{bF}$$

and

$$(a^2 - a)V_1^F = (a^2 - a)V_2^F$$

7.6.4 LL Fault

or

$$V_1^F = V_2^F$$

- Based on the equations,

$I_2^F = -I_1^F = \frac{1}{3}(a - a^2)I_i^{bF}$, and $V_1^F = V_2^F$, we can create a circuit as shown in the figure.

$$I_1^F = -I_2^F = \frac{1.0\angle 0°}{Z_1^{si} + Z_2^{si}}$$

and

$$\boldsymbol{I_i^{bF}} = \boldsymbol{I_0^F} + a^2\boldsymbol{I_1^F} + a\boldsymbol{I_2^F} = (a^2 - a)\boldsymbol{I_1^F} = -j\sqrt{3}\boldsymbol{I_1^F}$$

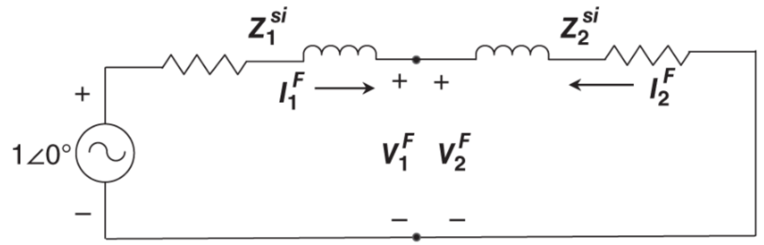


Fig: LL fault in sequence domain.

Thank You!